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Transparency and Price Formation

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Abstract

We study the role that price transparency plays in determining the efficiency and surplus division in a sequential bargaining model of price formation with asymmetric information. Under natural assumptions on type distributions, and for any discount factor, we show that the unobservability of past negotiations leads to lower prices and faster trading. The lack of transparency therefore enhances the "Coasian effect" by fostering efficiency and diverting more of the surplus to the player who possesses private information. In addition, we show that the equilibrium is unique and is in pure strategies in the non-transparent regime; this stands in sharp contrast to the existing literature and allows for a better understanding of the role of transparency.

Keywords: Price Formation, Transparency, Bargaining, Incomplete Information, Coase Conjecture

JEL codes: C61, C73, C78

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1 Introduction

Many markets operate largely in the absence of price transparency. For instance, in over-the-counter (OTC) or off-exchange derivatives trading, quotes and executions are confined within bilateral negotiations, and this information is not available to the rest of the market. Whether enhancing transparency via facilitating access to information about prices, negotiations, and transactions may improve efficiency in such markets has been a topic of debate in policy circles. Increased transparency may enhance efficiency by reducing search costs and fostering competition. However, in the presence of asymmetric information and dynamic trading—which are salient features of most markets of interest—potential informational spillovers across players and over time introduce additional channels through which transparency may influence the price formation process and hence market outcomes. In this paper we study a sequential bargaining model of price formation and isolate a channel via which increased transparency in this sense, indeed, unambiguously adversely affects market outcomes.

In our model, a buyer with unitary demand sequentially samples alternative sellers. Once the seller is sampled, a bargaining stage ensues that takes the form of a take-it-or-leave-it price offer by the seller. The buyer's sampling cost is captured by discounting. There is common knowledge of gains from trade, but the buyer has private information about his willingness to pay.

Formally, we amend the classic Coasian bargaining with sequential sellers in order to analyze two opposing information structures. Not only is sequential bargaining a workhorse in analyzing bilateral interactions, with applications ranging from dispute resolution to labor contracting, but it is also a theoretically important strategic price formation mechanism. In particular, bargaining between a long-run player and a sequence of short-run players is a building block of a dynamic search market where each time a participant can meet at most one possible trading partner.[1] Though it is stark compared to the intricacies of real markets, our sequential bargaining model well describes certain crucial aspects, for instance, of the OTC derivatives market in which "traders often search for attractive prices by sequentially contacting multiple counterparties. Once a quote is provided, the opportunity to accept quickly lapses" (Zhu, 2012). Therefore, we believe that the mechanism identified within this model is one of the forces driving price formation in real markets.

We analyze the role of price transparency in determining the surplus distribution (measured by the equilibrium prices), and the efficiency of trade (measured by the amount of delay before trade takes place). More specifically, we compare the equilibrium price sequences and expected delay under two opposing specifications: one where the past prices are observable to the ensuing

[1] See, e.g., Rogerson, Shimer, and Wright (2005).

sellers (*transparent regime*) and one where they are not (*non-transparent regime*). Under natural restrictions on the distribution of the buyer's valuations, we find that prices are uniformly lower in the non-transparent regime than in the transparent regime for each given discount factor of the long-run buyer. Moreover, even though an agreement is eventually reached in either regime, under further restrictions on the type distribution, the expected delay is larger in the transparent regime.

All of our results are obtained for arbitrary discount factors of the long-run buyer and not just for the case where the buyer is sufficiently patient. This feature makes our analysis of the effect of price transparency robust to the introduction of market frictions, which, beyond its theoretical interest, is valuable in understanding interactions in real markets where frictions cannot be ignored. Indeed, for the frictionless limit, the outcomes of both regimes are degenerate: trade is efficient and the informed player captures all the surplus. This is consistent with the classic Coase conjecture. From this perspective, our results imply that, away from the frictionless limit, a lack of transparency enhances the "Coasian effect" by fostering efficiency and diverting more of the surplus to the informed player. Moreover, the comparison with the Coase conjecture implies that transparency matters only in the presence of market frictions.

In an infinite horizon bargaining game without parametric assumptions or closed-form solutions, comparing equilibrium price paths in two different extensive forms is a rather challenging task. The breakthrough of this paper is made possible by observing that, from the elementary demand theory, the comparison of equilibrium prices in two markets boils down to the comparison of demand elasticities in these markets. We identify conditions ensuring an appropriate demand elasticity ranking for our two dynamic bargaining environments. Our appeal to demand theory not only resolves the analytical difficulties, but also highlights the economic forces at play in dynamic bargaining problems.

To gain some intuition, first recall the well-known "skimming property": regardless of the regime and in any equilibrium, a price is accepted by the buyer if and only if his valuation is above an associated cutoff. This property allows one to interpret the buyer's decisions as defining an endogenous demand curve that each seller faces in equilibrium, where the probability of trade at each price is interpreted as the quantity sold. Gul, Sonnenschein, and Wilson (1986) point out that, with this interpretation, when both parties are long-run, their bargaining problem can equivalently be viewed as the problem of a durable goods monopolist lacking the power to commit to a price. In contrast to Gul, Sonnenschein, and Wilson (1986) where a single durable goods monopolist competes with his future selves, in our model, a sequence of sellers compete with each other over time. Each seller faces a *residual market* characterized by a demand curve endogenously determined by the equilibrium strategies of all past and future sellers.

With this interpretation at hand, the crucial exercise is to compare the demand curves faced by each seller in both regimes. First consider a hypothetical price change by seller 1. Transparency forces seller 2, who enters the game only when there is no trade in the first period, to respond with a price change in the same direction. In contrast, in the non-transparent regime, seller 2 cannot react to such a price change. In equilibrium, the buyer fully anticipates the reaction of seller 2 and hence is less sensitive to the price change by seller 1 in the transparent regime. That is, the demand curve faced by seller 1 in the transparent regime is *steeper* than that in the non-transparent regime. However, the ranking of the slopes of the demand curves does not translate directly into the ranking of elasticities nor the ranking of profit-maximizing prices. Indeed, since a seller's demand curve is determined jointly by the strategic choices of all previous and future sellers, the relative positions of the two demand curves corresponding to the same seller in either regime is a priori unclear. Therefore, the above elementary intuition is not enough given the subtleties of our problem. We show that a relatively benign regularity condition on the buyer's type distribution—increasing hazard rate—pins down the relative positions of the demand curves and therefore allows an unambiguous comparison of prices in the two regimes.

We next explore which regime leads to a larger delay in trade. Delay in the context of the bargaining model is related to the quantity traded in the analogous dynamic monopoly model with larger quantities corresponding to smaller delay. Typically, a less elastic demand curve implies a higher price, but, as is well-understood in demand theory, elasticities alone do not determine the ranking of quantities. One needs to uncover additional details about the demand functions, which are endogenous equilibrium objects in our model. In spite of this, we are able to show that transparency entails more inefficient delay if the buyer's type distribution is concave.

Our contribution is not limited to the comparison of the two regimes. Even though the transparent regime in isolation is the focus of the Coase bargaining and durable goods monopoly literature, the equilibrium characterization of the non-transparent regime is novel to this paper. Remarkably, we are able to show that the equilibrium outcome in the non-transparent regime is unique and is necessarily in pure strategies under the minimal assumption of increasing virtual valuation. This result, which does not call forth any genericity assumption, is in sharp contrast to the classic results of Coase bargaining where offers are publicly observable, in which case, randomization in the first period and off the equilibrium path may be necessary (see Fudenberg, Levine, and Tirole (1985) and Ausubel, Cramton, and Deneckere (2002)). This pure strategy property is also surprising in view of the results pertaining to dynamic markets for lemons with unobservable offers where randomization is a generic property (see Hörner and Vieille (2009), and a concurrent paper by Fuchs, Öry, and Skrzypacz (2012)). This equilibrium property allows for a characterization of the role of transparency that is not possible elsewhere.

Related Literature

The role of observability has been previously investigated in different environments. Bagwell (1995) studies the connection between commitment power and observability and shows that the first-mover's advantage could be eliminated if its action is not perfectly observed. Rubinstein and Wolinsky (1990) study random matching and bargaining. In their complete information environment, observability enlarges the equilibrium set by a folk theorem argument that is not at work in the presence of incomplete information. Swinkels (1999) analyzes a dynamic Spencian signalling model and obtains pooling equilibrium under private offers; Nöldeke and van Damme (1990) previously obtained Riley outcome in the case of public offers.

Whereas we look at the more standard setup with independent valuations, another line of the literature studies bargaining with interdependent values; see the pioneering work by Evans (1989), Vincent (1989), and Deneckere and Liang (2006). More recently, Hörner and Vieille (2009) study an interdependent-value model with a single long-run player and a sequence of short-run buyers. They show that in the hidden-offer case, multiple equilibria exist, all in mixed strategies, and trade occurs with a delay even when the discount factor goes to 1, while in the public-offer case, remarkably, trade can only occur in the first period and an inefficient impasse ensues. The question about the impact of price transparency on surplus division and the timing of trade for general discount factors is not addressed, and they do not obtain a clear-cut comparison of the two regimes in terms of price paths and long-run player's welfare. It is noteworthy that our independent value model is not a limiting case of theirs, and hence, the qualitative divergence of results in terms of equilibrium structure, efficiency, and price comparisons, should come as no surprise and it illustrates the subtle role of the interdependent value assumption.[2] In a random matching model, Kim (2012) presents a case in which efficiency of trade may not be monotonic in the search friction.

In our model, efficiency is always obtained when discounting friction vanishes. We emphasize that bilateral sequential bargaining, rather than other centralized mechanisms, is an appropriate model for thin markets in which trading opportunities do not arise frequently and hence discounting frictions are non-negligible. Accordingly we focus on a comparison of price dynamics, surplus division and the timing of trade in the two regimes that is robust to all discounting frictions, and this task requires new methods. We obtain an unambiguous comparison

[2]The results of Hörner and Vieille (2009) rely crucially on the assumption that buyer-seller types are sufficiently interdependent and the discount factor is sufficiently large. Indeed, as the interdependence vanishes (i.e., the values of the uninformed short-run players become a constant), the lower bound required for the discount factor converges to 1, implying that the limiting case is not a well-defined Coasian bargaining game.

and show that the long-run buyer has a clear-cut preference over market information structures.

Several bargaining models that feature discounting as a source of search friction are similar in structure to our model. Fudenberg, Levine and Tirole (1987) consider bargaining games where a seller can decide whether to switch to a new buyer or continue to bargain with an incumbent buyer. They show that a take-it-or-leave-it offer endogenously emerges as an equilibrium outcome. Atakan (2008) studies bargaining situations where small parties can form a coalition to bargain with a common opponent; see also Segal (2003).

The paper is organized as follows: Section 2 introduces the formal model, Section 3 considers a two-period example to demonstrate the forces driving our results, Section 4 establishes the existence and the uniqueness of equilibrium for the non-transparent regime, Sections 5 and 6 present our results concerning the comparison of prices and speed of trade across two regimes, and Section 7 concludes. All omitted proofs are relegated to the Appendix.

2 Model

A long-run buyer bargains with a sequence of short-run sellers. In each period $t = 1, 2, ...,$ a new seller enters the game. We refer to the seller at period t as "seller t." Each seller has one unit to sell for which his reservation value is normalized to 0. The buyer has demand for one unit.

The buyer discounts future payoffs at rate $\delta \in (0, 1)$ and has private information about his valuation, v, which we refer to as his "type." The prior cumulative distribution of buyer types is F, which has support $[\underline{v}, \bar{v}]$, with $\bar{v} > \underline{v} > 0$, and admits density f.[3] We assume that there exists a constant $m > 0$ such that $\frac{1}{m} < f(v) < m$ for any $v \in [\underline{v}, \bar{v}]$. Throughout, we assume that F has increasing "virtual valuation"; that is,

$$v - \frac{1 - F(v)}{f(v)} \text{ is increasing.} \tag{1}$$

This assumption is standard in the mechanism design literature. Bulow and Roberts (1989) point out that this assumption is equivalent to the monotonicity of the marginal revenue curve of a monopolist seller facing buyers with type distribution F.

The bargaining within each period t is as follows: Seller t proposes a price p_t to the buyer. The buyer may choose to accept or reject this offer. If the price is accepted, the transaction takes place at this price and the bargaining game ends; the buyer obtains a payoff of $\delta^{t-1}(v - p_t)$, while seller t obtains a payoff of p_t. If the price is turned down, seller t leaves the market, and the game proceeds to period $t + 1$.

[3] That is, we focus on the so-called gap case. Please see Section 7 for a discussion of the role of this assumption.

The bargaining is prolonged only when previous prices are rejected. We refer to the information structure of the game where past rejected offers are observable (respectively, unobservable) to the subsequent sellers as the "transparent regime" (respectively, the "non-transparent regime").

We consider the perfect Bayesian equilibria of the two specifications of the bargaining game. The first thing to notice is that in both regimes, the "skimming property" is satisfied. That is, after any history, on or off the equilibrium path, if a price offer p is accepted by a type v, then it is also accepted by all types $v' > v$. This allows us to cast the problem of each seller as that of choosing a cutoff type k to trade with (or a probability of trade) rather than choosing a price.[4]

In the equivalent dynamic monopoly interpretation of the model in the spirit of Gul, Sonnenschein, and Wilson (1986), a sequence of sellers, each with unlimited supplies, face a continuum of long-run buyers distributed over $[\underline{v}, \bar{v}]$. Each seller could serve a fraction of the market at some transaction price. The game is prolonged because the market is not fully penetrated, not because all previous prices are rejected (in each period some prices could be offered and accepted).

3 An Example

We first consider a two-period version of our model. For further simplicity, we assume that buyer types are uniformly distributed with support $[0, 1]$.[5]

By the skimming property, for each on or off-the-equilibrium-path history, there exists k_1 such that seller 2 believes that buyer types higher than k_1 trade with seller 1 and the remaining types are $[0, k_1]$. Since the second period is the final period, regardless of the regime, a remaining buyer type k accepts seller 2's offer p_2 if p_2 is below k. Therefore, seller 2's problem in either regime can be cast as choosing $p_2 = k$ to solve:

$$\max_k k(k_1 - k).$$

Then, regardless of the regime, when the remaining types are $[0, k_1]$, seller 2 charges a price $p_2 = \frac{k_1}{2}$ and trades with buyer types $\left[\frac{k_1}{2}, k_1\right]$.

[4]See, e.g., Fudenberg and Tirole (1991, p. 406). The proof for the skimming property does not rely on the assumption of observability; the crucial elements are price posting by the seller and single-unit demand by the buyer. In the non-transparent regime, it can be shown that the buyer uses a reservation price strategy, which is stronger than the skimming property.

[5]Even though in our general model we assume $\underline{v} > 0$, with finite horizon the specification of $\underline{v} = 0$ does not affect the qualitative results, yet it simplifies the computation.

The two regimes differ in the formation of beliefs off the equilibrium path: whereas an off-path price of seller 1 in the non-transparent regime does not affect the belief of seller 2, who cannot observe this deviation, it does so in the transparent regime. To be more specific, in the non-transparent regime, seller 2 believes that the highest remaining buyer type is a *fixed constant* k_1^*, even when the actual cutoff of seller 1 is different. Therefore seller 2's price in the second period is a *fixed constant* equal to $\frac{k_1^*}{2}$. If seller 1 wishes to sell to types $[k, 1]$, the highest price he can charge is

$$p_1(k) = (1-\delta)k + \delta\frac{k_1^*}{2}. \tag{2}$$

This is the price that makes the marginal type k indifferent between buying at a price $p_1(k)$ now and waiting until the second period for the constant price $\frac{k_1^*}{2}$. Hence, seller 1 in the non-transparent regime solves the following problem:

$$\max_k (1-k)p_1(k). \tag{3}$$

It follows that $k_1^* = \frac{1}{2}\left(1 - \frac{1}{2}\frac{\delta}{1-\delta}k_1^*\right)$. Hence,

$$k_1^* = \frac{1}{2}\left(1 - \frac{\delta}{4-3\delta}\right),\ p_1^* = \left(\frac{1}{2} - \frac{\delta}{4}\right)\left(1 - \frac{\delta}{4-3\delta}\right),\ p_2^* = \frac{1}{4}\left(1 - \frac{\delta}{4-3\delta}\right).$$

In contrast, in the transparent regime, if seller 1 sells to buyer types $[k, 1]$ for any k, seller 2 will correctly anticipate the remaining types to be $[0, k]$ and charge a price $\frac{k}{2}$ accordingly. Moreover, seller 2's response of setting price $\frac{k}{2}$ to seller 1's deviation is fully anticipated by the buyer, implying that the highest price that seller 1 can charge and sell to buyer types $[k, 1]$ is

$$p_1(k) = (1-\delta)k + \delta\frac{k}{2} = k\left(1 - \frac{\delta}{2}\right). \tag{4}$$

Now, seller 1's problem is given by (3) where $p_1(k)$ is specified by (4). Simple algebra shows that

$$k_1^* = \frac{1}{2},\ p_1^* = \frac{1}{2}\left(1 - \frac{\delta}{2}\right),\ p_2^* = \frac{1}{4}.$$

We summarize our finding in Table 1 below.

	period 1 cutoff	period 2 cutoff	period 1 price	period 2 price
non-transparent	$\frac{1}{2}\left(1 - \frac{\delta}{4-3\delta}\right)$	$\frac{1}{4}\left(1 - \frac{\delta}{4-3\delta}\right)$	$\left(\frac{1}{2} - \frac{\delta}{4}\right)\left(1 - \frac{\delta}{4-3\delta}\right)$	$\frac{1}{4}\left(1 - \frac{\delta}{4-3\delta}\right)$
transparent	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2} - \frac{\delta}{4}$	$\frac{1}{4}$

Table 1: Comparison of the two regimes

The surprising contrast becomes more apparent if we take $\delta \to 1$ as illustrated in Table 2 below.

	period 1 cutoff	period 2 cutoff	period 1 price	period 2 price
non-transparent	0	0	0	0
transparent	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Table 2: Comparison of the two regimes as $\delta \to 1$

This example illustrates the following qualitative results that we generalize later. The prices are uniformly higher in the *transparent* regime, and hence, the lack of transparency diverts more surplus to the informed long-run buyer. In addition, the expected delay in trade; i.e., the expected value of $1 - \delta^{\tau(k)}$ where $\tau(k)$ is the period in which type k trades is higher in the *non-transparent* regime, and hence, the lack of transparency fosters efficiency.

4 Equilibrium

The analysis of the transparent regime follows from Fudenberg, Levine, and Tirole (1985) and Gul, Sonnenschein, and Wilson (1986). Their results in fact apply to two bargainers with unequal discount factors – including the case in our sequential search model, where the sequence of short-run sellers can effectively be thought of as one seller with discount factor equal to 0.[6] We now turn to the more interesting non-transparent regime. We establish that there is a unique equilibrium that is in pure strategies in this regime.

Theorem 1 *Fix any $\delta \in (0, 1)$. The equilibrium in the non-transparent regime exists and is unique. In addition, there exists $0 < T < \infty$, such that all buyer types trade with probability 1 within T periods, and all players use pure strategies at or before period T.*

This result is in contrast with existing results in two strands of the literature. In Coasian bargaining models, uniqueness is established under a genericity condition; moreover, randomization is often required off the equilibrium path: pure strategy equilibria have not been established except for specific examples that allow for closed-form solutions, such as the uniform distribution we analyzed in the example above; see, e.g., Stokey (1981) and Sobel and Takahashi (1983).[7]

[6]It is known in this literature that the equilibrium is unique for generic type distributions, and trade takes place with probability 1 within a finite number of periods. In addition, even though in this regime the equilibrium need not be in pure strategies, on the equilibrium path randomization can occur only in the first period. However, when characterizing off-equilibrium play, randomization could be necessary even with short-run sellers.

[7]Note that when seller $T + 1$ is approached by the buyer, the seller's belief could be arbitrary in a perfect Bayesian equilibrium, and hence multiple off-path play could be supported in period $T + 1$. However, this does not affect the equilibrium outcome. The Coasian bargaining literature does not consider this kind of multiplicity of off-path play.

In bargaining models with interdependent values, typically no pure strategy equilibrium exists; see, e.g., Hörner and Vieille (2009).

The unobservability of price history entails two competing effects that are absent in the transparent regime. On the one hand, the skimming property implies only that the posterior beliefs are *distributions over right truncations of the prior*—instead of simple right truncations of the prior, as would be the case if the history were observable. This is simply because the outcomes of potential randomizations by previous sellers are not observable (except trivially in the first period when there is no prior randomization). On the other hand, if the posterior beliefs were indeed simple right truncations of the prior, our assumption of increasing virtual valuation (or equivalently, the decreasing marginal revenue property) would imply that each seller has a unique optimal pure strategy in the non-transparent regime, since in this regime the "inverse demand curve" faced by the analogous monopolist is simply a linear transformation of $1 - F$. This is in contrast to the transparent regime, where the demand faced by each seller must take into account the reaction of subsequent sellers, which depends on the details of F. Therefore, the crux of our proofs for pure strategy and uniqueness is to show that the posterior beliefs, even when price history is not observable, are necessarily simple right truncations of the prior.

The complete proof for Theorem 1 in Appendix A is quite involved. Our idea of establishing pure strategy is to successively narrow down the supports of mixed strategies, which we believe is interesting and may be of use elsewhere. We offer a brief outline of our proof for interested readers. Fix an equilibrium and let T be the last period in which trade occurs with a positive probability in that equilibrium.[8] As mentioned above, by the skimming property, we can identify seller t's offer p_t in period t with the *marginal* buyer type k_t, the lowest type that will accept the price p_t. Since seller t can play mixed strategies, the marginal types can be random as well. Let K_t denote the support of marginal types in seller t's randomization. In the fixed equilibrium of the non-transparent regime, K_t only depends on the calendar time t, not on the realizations of previous price offers. Write $\bar{k}_t = \sup K_t$ as the supremum of the support of seller t's randomization. Our goal is to show that for each $0 < t \leq T$, $K_t = \left\{\bar{k}_t\right\}$ and hence establish that all equilibrium strategies at or before T must be pure. The next lemma is the critical step toward establishing this result.

Lemma 1 *For any* $\tau = 1, ..., T-1$, $(\cup_{t=1}^{\tau} K_t) \cap [\bar{k}_{\tau+1}, \bar{k}_1] = \left\{\bar{k}_1, \bar{k}_2 ..., \bar{k}_\tau\right\}$.

Proof: See Appendix A.2. ■

[8] Note that the number of periods it takes for the game to end depends on δ, and it grows unboundedly as $\delta \to 1$.

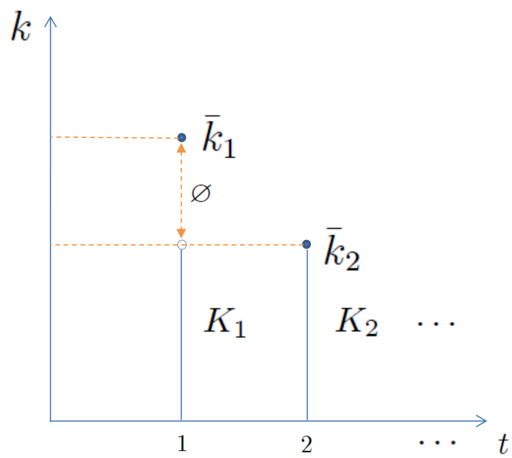

Figure 1: K_t is the support of seller t's randomization. Lemma 1 implies that K_1 does not intersect with $[\bar{k}_2, \bar{k}_1)$, and hence, the support of seller 1's randomization is narrowed down.

Figure 1 illustrates the content of this lemma for $\tau = 1$. The lemma establishes that, for any $t < T$, all but the highest cutoff types in the support of seller t's randomization are smaller than the supremum of the support of seller T's randomization, $\bar{k}_T$. Then, since all trade must take place in T periods, and hence $\bar{k}_T = \underline{v}$, there are no cutoffs but $\bar{k}_t$ in the support of seller t's randomization, which completes the argument for the pure strategy.

In a pure strategy equilibrium, the posterior type distribution must always be a truncation of the prior distribution F. Then the seller who faces the highest remaining type k solves the following profit-maximization problem:

$$\max_{k'} \left(F(k) - F(k')\right) \left[(1-\delta)k' + \delta p\right].$$

The assumption of increasing virtual valuation guarantees that whenever the continuation equilibrium price p is less than k, there is a unique solution k' to the seller's problem. In contrast, in the transparent regime, since the continuation price p depends on today's choice k', the uniqueness of the solution to the seller's profit-maximization problem is not guaranteed. This distinction allows us to establish the uniqueness of equilibrium in the non-transparent regime, in contrast to the transparent regime.

5 Price Comparison

In this section we make the stronger assumption that F exhibits an increasing hazard rate; i.e., $\frac{f(v)}{1-F(v)}$ is non-decreasing over $[\underline{v}, \bar{v}]$. This assumption is introduced into the bargaining setup by

Ausubel and Deneckere (1993), and we uncover a crucial connection between this assumption and demand theory. Under this assumption, we establish that any equilibrium realization of the price sequence in the transparent regime is uniformly above that in the non-transparent regime.

Let $i = TR, NTR$ indicate the transparent and non-transparent regimes, respectively. We let $\{p_t^i\}$ represent a *realized* equilibrium price sequence in regime i and let T^i be the last period in which the trade takes place with positive probability along this equilibrium path. The price comparison will not be well-defined if $T^{TR} \neq T^{NTR}$. To overcome this, we adopt the convention that $p_t^i = \underline{v}$ for $t > T^i$.

Theorem 2 *Fix any* $\delta \in (0,1)$. *Suppose that* F *exhibits an increasing hazard rate. Let* $\{p_t^{TR}\}$ *be any realization of the equilibrium price sequence in the transparent regime, and let* $\{p_t^{NTR}\}$ *be the unique equilibrium price sequence in the non-transparent regime. Then* $p_t^{TR} \geq p_t^{NTR}$ *for all* t.

The complete proof is relegated to Appendix B. Here we provide an outline to help readers navigate through the proof and explain how the increasing hazard rate assumption is used. First note that the result will be vacuously true if the buyer type distribution F is such that in either regime all trade takes place within the first period, in which case the transaction prices are identically $\underline{v}$, the lowest buyer type. So suppose this is not the case. We first show that the ranking holds for the first period, i.e., $p_1^{TR} \geq p_1^{NTR}$.[9] The crucial intuition is discerned by comparing the "demand curves" that seller 1 in either regime faces.

Our proof amounts to showing that the demand curve in the non-transparent regime is *more elastic* than that in the transparent regime at p_1^{TR} – the equilibrium price offer of the transparent regime. Then, it follows from the elementary monopoly-pricing theory that the first-period profit-maximizing price in the non-transparent regime is *lower* than p_1^{TR}.

Let $p_2^{TR}(p)$ be the equilibrium second period price in the transparent regime, as a function of the first period price p.[10] Then, for any first period price p, the cutoff buyer types who purchase in the first period in the two regimes – $k_1^{TR}(p)$ and $k_1^{NTR}(p)$ – are determined by the following indifference conditions:

$$p = (1-\delta)k_1^{TR}(p) + \delta p_2^{TR}(p) \tag{5}$$

[9] However, note that after the first period, the posterior belief in the two regimes might be different. Hence the price ranking from period 2 on is not immediate. See Appendix B.2, Lemma 15 for the complete argument.

[10] Here, in order to give a clean intuition, we have focused on the case where all sellers make pure strategy price offers after any history. As discussed previously, this may not be the case in an equilibrium of the transparent regime. The Appendix B deals with the general case.

and

$$p = (1-\delta)k_1^{NTR}(p) + \delta p_2^{NTR} \tag{6}$$

which can be re-arranged, respectively as follows:

$$k_1^{TR}(p) = \frac{p - \delta p_2^{TR}(p)}{1-\delta} \qquad \text{and} \qquad k_1^{NTR}(p) = \frac{p - \delta p_2^{NTR}}{1-\delta} \tag{7}$$

As we explained earlier, seller 1's problem in either regime can be thought of as the problem of a monopolist who faces a demand curve that is shaped by the strategies of the subsequent sellers, by identifying the probability of trade with the quantity sold. In other words, by letting $Q = 1 - F(k)$ stand for the quantity sold, the above indifference conditions can be interpreted as the inverse demand curves faced by seller 1 in either regime. Then the key is to compare the following two demand curves faced by seller 1 in the two regimes:

$$Q^{TR}(p) = 1 - F(k_1^{TR}(p))$$

and

$$Q^{NTR}(p) = 1 - F(k_1^{NTR}(p)).$$

Letting p_1^i represent the first period equilibrium price in regime i, and assuming *for the purpose of contradiction* that $p_1^{TR} < p_1^{NTR}$, we argue for the following:

$$\frac{Q^{TR}(p_1^{TR}) - Q^{TR}(p_1^{NTR})}{Q^{TR}(p_1^{TR})} \leq \frac{Q^{NTR}(p_1^{TR}) - Q^{NTR}(p_1^{NTR})}{Q^{NTR}(p_1^{TR})}, \tag{8}$$

that is, the percentage decline in the quantity sold in response to a price increase from p_1^{TR} to p_1^{NTR} is smaller in the transparent regime than in the non-transparent regime. Our argument has three steps.

First, note that

$$\Delta^{NTR} \equiv k_1^{NTR}(p_1^{NTR}) - k_1^{NTR}(p_1^{TR}) \geq k_1^{TR}(p_1^{NTR}) - k_1^{TR}(p_1^{TR}) \equiv \Delta^{TR}, \tag{9}$$

that is, the size of the interval of types purchasing in the first period *shrinks more* in the non-transparent regime than in the transparent regime following a price increase from p_1^{TR} to p_1^{NTR}. This simply follows from (7) and the fact that the second period price in the transparent regime, $p_2^{TR}(p)$, is non-decreasing in the first period price p.[11] Intuitively, this is because any price change by seller 1 in the transparent regime elicits a response by seller 2 in the form of a price change in the same direction, which is anticipated by the buyer. In particular, a price

[11]This intuitive assertion does require a proof: it follows because the first period marginal type is non-decreasing in the first period price (Lemma 11 in the Appendix) and the second period price is non-decreasing in the first period marginal type (Lemma 10 in the Appendix).

increase in the transparent regime implies higher prices in ensuing periods, so that, following such an increase, a smaller range of types switch from buying to not buying when compared with the case of the non-transparent regime – where a price change cannot be matched by the ensuing sellers.

Second, we posit that

$$k_1^{TR}(p_1^{TR}) \leq k_1^{NTR}(p_1^{TR}), \tag{10}$$

or equivalently, $Q^{TR}(p_1^{TR}) \geq Q_1^{NTR}(p_1^{TR})$. In words, the monopolist corresponding to the transparent regime sells more at price p_1^{TR} than would the monopolist corresponding to the non-transparent regime.[12]

Third, we note that the *increasing hazard rate property* of the type distribution F implies that the quantity $\frac{F(k+\Delta)-F(k)}{1-F(k)}$ is increasing in k. This quantity is also increasing in Δ by the monotonicity of F. To use this observation, we note that

$$\frac{Q^i(p_1^{TR}) - Q^i(p_1^{NTR})}{Q^i(p_1^{TR})} = \frac{F(k^i(p_1^{TR}) + \Delta^i) - F(k_1^i(p_1^{TR}))}{1 - F(k_1^i(p_1^{TR}))}, i = TR, NTR, \tag{11}$$

where Δ^i is defined in (9). Then (8) follows because $\Delta^{TR} \leq \Delta^{NTR}$ (by (9)) and $k_1^{TR}(p_1^{TR}) \leq k_1^{NTR}(p_1^{TR})$ (by (10)).

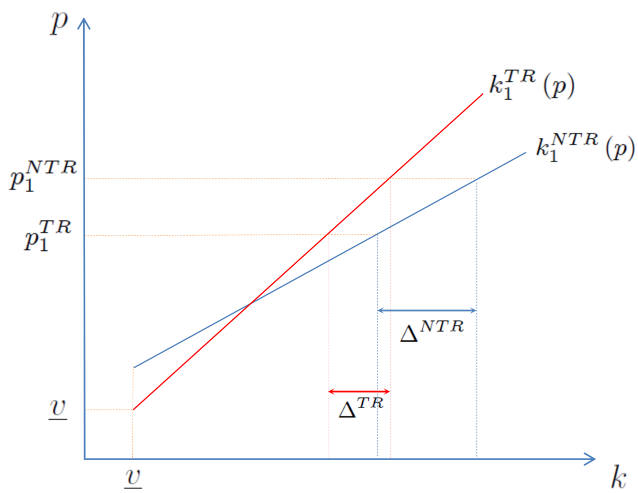

Figure 2: The role of the increasing hazard rate assumption.

Figure 2 depicts the mapping between the price choice and the marginal type who would choose to trade at that price for each regime. The distinction of having observable versus

[12]Our argument for this assertion requires the use of induction. Intuitively, if the second period equilibrium prices in the transparent regime are higher than those in the non-transparent regime (Lemma 13 in the Appendix), then, since in either regime the first period price is simply a weighted average of the marginal type who buys at that price and the second period price (see (5), (6)), the marginal type that buys at price p_1^{TR} is larger in the non-transparent regime.

unobservable prices guarantees that Δ^{NTR} is greater than Δ^{TR}. However, since the intervals $[k_1^i(p_1^{NTR}) - k_1^i(p_1^{TR})]$, $i = TR, NTR$, are in general not nested, their relative measure under F cannot be ranked. The increasing hazard rate property guarantees that this measure relative to the measure of $[\underline{v}, k_1^i(p_1^{NTR})]$ is larger in the non-transparent regime.

To summarize, (8) means that the percentage change in quantity in response to a given price change (from p_1^{TR} to p_1^{NTR}) is larger in the non-transparent regime: i.e., at this range of prices, the demand curve faced by the monopolist in the non-transparent regime is more elastic. Nevertheless, this monopolist strictly prefers the higher price p_1^{NTR} to the lower price p_1^{TR}, since p_1^{NTR} is the unique solution to his profit maximization problem. Then, the monopolist in the transparent regime, facing a less elastic demand, should also have the same preference over these two prices, which contradicts the optimality of p_1^{TR}. This establishes that $p_1^{TR} \geq p_1^{NTR}$.

Since the price paths in the two regimes that the long-run buyer faces are uniformly ranked, it follows immediately that the buyer has a clear-cut preference over the two regimes.

Corollary 1 *Fix any $\delta \in (0, 1)$. Suppose that F exhibits an increasing hazard rate. Then the long-run buyer is better off in the non-transparent regime.*

6 Expected Delay

As is well-understood in Coasian dynamics, as the buyer becomes extremely patient, the outcome becomes efficient. However, the literature so far has little to say about the delay and efficiency when δ is bounded away from 1. Instead, the literature has focused on the limiting case of $\delta \to 1$ to study "real delay" in various environments. Studying the equilibrium outcomes in the limiting case is not only conceptually important to our understanding of commitment power but also facilitates definite conclusions, such as the limiting efficiency result for the gap case. However, to understand fully the applications in the real market environments, it is necessary to also consider discount factors that are bounded away from 1. This section is concerned with the question of which regime leads to a longer delay in trade for *any* buyer discount factor $\delta \in (0, 1)$.

Under our interpretation, which identifies the probability of sale with the quantity sold by a residual monopolist, the expected delay in sale is smaller if the sales are more "front-loaded"—i.e., if earlier sellers cover a larger share of the market. In the example of Section 3, comparison of the two regimes in this dimension was immediate from the fact that the first seller in the non-transparent regime serves a larger share of the market (targets a smaller cutoff type). In the general model, however, the comparison of quantities sold by seller 1—or subsequent sellers, for that matter—in the two regimes is not possible and should not be expected. This is because demand curves faced by seller 1 in either regime typically are related to each other in the manner shown in Figure 3. Therefore, even though the ranking of the elasticities is possible, it only implies that seller 1 in the transparent regime chooses a price above p_1^{NTR}, and not necessarily above $\bar{p}$. Nevertheless, we are able to establish the result for the general model under the additional assumption that the buyer type-distribution is concave.

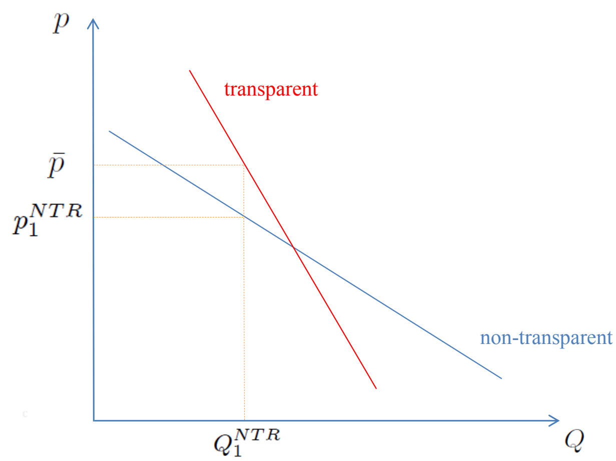

Figure 3: Representative demand curves of seller 1 for the infinite horizon model. Note that the two demand curves intersect at a price lower than the first period equilibrium price of the unobservable regime. This is because when the horizon is longer than two periods, the game starting in the second period onward is no longer identical across the two regimes.

To formally state our result, let $\{k_t^{TR}\}$ be a *realization* of equilibrium cutoff levels in the transparent regime and let $\{k_t^{NTR}\}$ be the unique equilibrium cutoff sequence in the non-transparent regime with the convention that $k_t^i = \underline{v}$ for $t > T^i$ where T^i is the last period such that trade takes place with positive probability along the realized path. Given these sequences, for each type $v < \bar{v}$, and for either $i = NTR, TR$, there is a unique t, such that $k_t^i > v \geq k_{t+1}^i$, with the convention that $k_0^i = \bar{v}$. Let $t^i(k)$ represent this t.

Then, a measure of the delay that type v experiences is $1 - \delta^{t^i(v)}$, which is the portion of the payoff lost due to the delay in reaching an agreement. Therefore, the expected delay in regime $i = TR, NTR$ is

$$\int_{\underline{v}}^{\bar{v}} (1 - \delta^{t^i(v)}) dF(v) = 1 - \int_{\underline{v}}^{\bar{v}} \delta^{t^i(v)} dF(v).$$

Notice that ex ante, the probability that the trade will take place at period t is $F(k_{t-1}^i) - F(k_t^i)$. Therefore, the above expectation can alternatively be expressed as

$$1 - \sum_{t=1}^{\infty} \delta^{t-1} (F(k_{t-1}^i) - F(k_t^i))$$

which simplifies to

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}F(k_t^i). \tag{12}$$

Theorem 3 *Fix any $\delta \in (0,1)$. Assume that $p_t^{TR} \geq p_t^{NTR}$ for any t where $\{p_t^{TR}\}$ is any realization of equilibrium prices in the transparent regime and $\{p_t^{NTR}\}$ is the unique equilibrium sequence of prices in the non-transparent regime. Then, for concave F, the expected delay in the transparent regime is larger than the expected delay in the non-transparent regime.*

Proof: See Appendix C. ■

To gain some intuition into how the ranking of prices helps and what role the concavity of type distribution F plays, note that for each $i = TR, NTR$, we have[13]

$$p_t^{TR} = (1-\delta)\sum_{l=t}^{\infty}\delta^{l-t}k_l^{TR} \geq (1-\delta)\sum_{l=t}^{\infty}\delta^{l-t}k_l^{NTR} = p_t^{NTR}. \tag{13}$$

In words, the discounted sum of the tails of the cutoff sequence from period t on (which is equal to the price in that period) is larger for the transparent regime than for the non-transparent regime. It is clear that when F is applied to each k_t^i to obtain the expression for the expected delay in (12), this ranking need not be preserved. The substance of Theorem 3 is to show that this ranking is preserved when F is concave. The intuition can most easily be gleaned from the following thought experiment: suppose in each regime trade takes place in the second period at the latest. Then, (13) implies that

$$(1-\delta)k_1^{NTR} + \delta k_2^{NTR} \leq (1-\delta)k_1^{TR} + \delta k_2^{TR} \quad \text{and} \quad k_2^{NTR} \leq k_2^{TR}.$$

This means that one of the following two rankings must hold:

1. $k_1^{NTR} \leq k_1^{TR}$ and $k_2^{NTR} \leq k_2^{TR}$;
2. $k_1^{NTR} > k_1^{TR} \geq k_2^{TR} \geq k_2^{NTR}$.

If the ranking in 1 obtains, (12) follows immediately from the monotonicity of F without referring to concavity. Under the ranking in 2, the cutoffs in the transparent regime are "less spread-out"—as well as on average higher—than those in the non-transparent regime, which

[13] That the prices are discounted sums of the cutoff types follows from the cutoff buyer type's indifference condition

$$p_t^i = (1-\delta)k_t^i + \delta p_{t+1}^i$$

for $i = TR, NTR$. Also since we are only concerned with equilibrium path cutoff types, even in the observable regime, future cutoffs (and therefore prices) are deterministic.

implies that, evaluated under a concave and increasing function F, their expectation will be larger.

To see how this intuition is generalized to longer horizons, consider an alternative interpretation of our model as a bargaining game with a stochastic deadline: suppose that δ, instead of representing the discount factor of the buyer, represents the probability with which the bargaining ends before the next period, conditional on the fact that it has not so far ended. It is well-understood that such a game is strategically equivalent to the game we have analyzed so far. Then, "the smallest buyer type that gets to trade" in either regime is a random variable assigning probability δ^{t-1} to k_t^i. If the realization of this random variable is k, then the realized probability (or quantity) of sale is $1 - F(k)$. With this interpretation, 1 minus the expression in (12) is the expected quantity sold or equivalently the expected probability of sale in either regime. What (13) allows us to show is that the random variable determining the smallest buyer type that trades in the transparent regime *second order stochastically dominates* its non-transparent regime counterpart. That is why the expectation of this random variable evaluated at concave F is larger in the *transparent* regime, implying a lower expected probability of trade or, equivalently, higher expected delay when past rejected prices are observable.

7 Concluding Remarks

To conclude, we emphasize several aspects of our model that we feel deserve further elaboration.

We can draw parallels between our model and well-known models of oligopoly. In particular, the bargaining model where previous prices are observable to future sellers is reminiscent of the "Stackelberg competition" in oligopoly, as later sellers observe and react to choices that earlier sellers have "committed to." The non-transparent regime instead is suggestive of the "Cournot competition" because each seller makes a choice without observing the choices of any other seller. However, the analogy is not complete because of two distinctions: first, our sellers compete in prices, which are strategic complements, rather than in quantities, which are strategic substitutes. Second, unlike in the standard theory of homogeneous good oligopolies, the products sold by different sellers in our model are vertically differentiated. The reason is that, in either regime, for a given price, each type of buyer prefers to buy from an earlier seller rather than wait for a later seller because of discounting. This wedge created by discounting would be analogous to the specification of vertically differentiated products with earlier sellers in the bargaining model corresponding to the sellers of a higher quality product in an analogous oligopoly model. Then the comparison of the outcomes of the bargaining games with observable versus unobservable price histories is analogous to the comparison of the outcomes of sequential versus simultaneous price (or quantity) setting in an oligopoly market with vertically differentiated products.

Another "competitive" benchmark with which we can contrast the performance of the two regimes is one where there is "within-period competition" between the short-run sellers; i.e., the buyer meets two (or more) sellers each period and price is determined by competitive bidding. In this case, it is easy to see that the prices will immediately be zero in either regime. Therefore, our results suggest that the non-transparent regime leads to outcomes that are "closer" to this benchmark. Rather than studying within-period competitive bidding, which would more reasonably describe a "thick" market, our purpose in studying one-to-one bargaining is precisely to unlock the strategic aspect of price formation.

Our bargaining model corresponds to the "gap" case—well-known in the bargaining literature. In fact, the Coase conjecture fails in the no-gap case with short-run sellers: We can employ the insights of Ausubel and Deneckere (1989) to construct multiple non-stationary equilibria, where the long-run buyer builds a "reputation" for having a low willingness to pay. Nevertheless, the gap case captures cleanly the critical insights of imperfect competition by silencing the reputation effect.

Our model features one-sided incomplete information and the uninformed party making offers. An interesting research direction is to see the role of price transparency in bargaining environments with two-sided uncertainty and bilateral offers. The major complication is of course the signalling aspect; see, for example, Cramton (1984), Chatterjee and Samuelson (1987), and Abreu and Gul (2000).

Appendix

A Proof of Theorem 1

A.1 Preliminary Results

In our dynamic environment, the distribution of types varies over time. We first make an observation regarding an implication of the monotone virtual valuation property for truncated distributions.

Lemma 2 *Assume F has an increasing virtual valuation: $k - \frac{1-F(k)}{f(k)}$ is strictly increasing. Then $k - \frac{\alpha - F(k)}{f(k)}$ is strictly increasing in k whenever $F(k) < \alpha \leq 1$.*

Proof: Consider $k' < k$ and $F(k) < \alpha$. We want to show

$$k - \frac{\alpha - F(k)}{f(k)} > k' - \frac{\alpha - F(k')}{f(k')}. \tag{14}$$

Define

$$L(\alpha) = k - k' + \frac{F(k) - F(k')}{f(k')} - (\alpha - F(k)) \left[\frac{1}{f(k)} - \frac{1}{f(k')} \right].$$

(14) is equivalent to $L(\alpha) > 0$. Notice that $L(1) > 0$ since F has increasing virtual valuation. For $\alpha < 1$, we have two cases to consider: if $\frac{1}{f(k)} - \frac{1}{f(k')} \leq 0$, then $L(\alpha) > 0$ follows immediately by the definition of $L(\alpha)$; if $\frac{1}{f(k)} - \frac{1}{f(k')} > 0$, then $L(\alpha)$ is decreasing in α, and hence $L(\alpha) > L(1) > 0$. ■

By virtue of the "skimming property," we shall identify price p with the infimum buyer type who accepts p. Since the seller in period t could potentially randomize, let K_t be the support of the cutoffs in period t and write $K = \cup_{t=1}^{T} K_t$. For each t, let $\bar{k}_t = \sup K_t$ and $\underline{k}_t = \inf K_t$. Hence after period t, the largest possible interval of remaining types is $[\underline{v}, \bar{k}_t)$, while the smallest such interval is $[\underline{v}, \underline{k}_t)$. Define $\bar{k}'_t = \sup K_t \backslash \{\bar{k}_t\}$. By convention, $\sup \varnothing = -\infty$. Therefore, $[\underline{v}, \bar{k}'_t)$ is the second largest possible interval after a (potential) seller randomization in period t. Note that it is possible a priori that $\bar{k}'_t = \bar{k}_t$. By standard arguments, $\underline{k}_t \geq \underline{v}$ for any t, and in any equilibrium, the game ends in finite time with a price equal to $\underline{v}$. This is formalized in Lemma 3.

Lemma 3 *In any equilibrium of the non-transparent regime, there exists $T > 0$ such that trade takes place with probability 1 within T periods.*

Proof: *Step 1:* Note that a seller never makes a price offer below $\underline{v}$. The argument is standard: all buyer types will accept a price of $(1 - \delta)\underline{v}$, which is better than waiting for a price of 0 tomorrow; but then $(1 - \delta^2)\underline{v}$ will be accepted for sure because the best price tomorrow will be $(1 - \delta)\underline{v}$; iterating this argument shows that a seller will never make a price offer below $(1 - \delta^n)\underline{v}$ for any n, and the claim follows.

Step 2: Suppose to the contrary there is an equilibrium in which some positive measure of types never trade for some history. Then in this equilibrium, $\overline{k}_t > \underline{v}$ for any $t > 0$. Clearly, $\{\overline{k}_t\}$ is a decreasing and hence convergent sequence. Consequently, $|\overline{k}_t - \overline{k}_{t+1}| \to 0$. Thus, the profit of seller t converges to 0 as $t \to \infty$. Moreover, it must be that $\overline{k}_t \downarrow \underline{v}$ because if $\lim_{t \to \infty} \overline{k}_t = k^* > \underline{v}$, then any seller could deviate to charge a price $\underline{v}$, which, by the previous claim, guarantees a strictly positive profit $(F(k^*) - F(\underline{v}))\underline{v}$, a contradiction.

Step 3: Now from Step 2, for each $\varepsilon > 0$, there exists t such that $\underline{v} < \overline{k}_t < \underline{v} + \varepsilon$. Then we claim that there exists ε such that for any $k \in (\underline{v}, \underline{v} + \varepsilon)$ and any $k' \in (\underline{v}, k]$,

$$(F(k) - F(k'))\, k' < F(k)\, \underline{v}. \tag{15}$$

To see this, note that the left-hand side is differentiable in k', and its derivative is $-f(k')k' + F(k) - F(k')$. Note that

$$\begin{aligned}
-f(k')k' + F(k) - F(k') &< -\frac{1}{m}\underline{v} + F(k) - F(k') \\
&< -\frac{1}{m}\underline{v} + F(\underline{v}+\varepsilon) - F(\underline{v}) \\
&< -\frac{1}{m}\underline{v} + m\varepsilon.
\end{aligned}$$

Hence, when $\varepsilon < \frac{1}{m^2}\underline{v}$, $-f(k')k' + F(k) - F(k') < 0$, and (15) follows immediately.

Step 4: Notice that the left-hand side of (15) is the highest possible payoff a seller could obtain when facing buyer types $[\underline{v}, k]$ if he wants to sell to the types $[k', k]$ (it assumes that a price equal to k' will be accepted by all types above k'), while the right-hand side of (15), by Step 1, is the seller's exact payoff by making a price offer $\underline{v}$. Therefore, (15) implies that if $\overline{k}_t < \underline{v} + \frac{1}{m^2}\underline{v}$, it is an ex post strictly dominant strategy for seller $t+1$ to make a price offer equal to $\underline{v}$ for each realization of $k_t \in (\underline{v}, \overline{k}_t]$. Therefore, $k_{t+1} = \underline{v}$ is an ex-ante strictly dominant strategy for seller t as long as $\underline{v} < \overline{k}_t < \underline{v} + \frac{1}{m^2}\underline{v}$. This contradicts the supposition that $\overline{k}_{t+1} > \underline{v}$ for each t. ■

We next argue that the upper bound of the support of a seller's potential randomization is strictly decreasing over time periods during which trade takes place with positive probability.

Lemma 4 *In any equilibrium in which the game ends for sure at T, we have $\bar{v} > \bar{k}_t > \bar{k}_{t+1}$ for any $t < T$.*

Proof: If $\bar{k}_t \leq \bar{k}_{t+1}$, then seller $t+1$ gets 0 profit. He could get positive profit by charging $\underline{v}$. Moreover, if $\bar{k}_t = \bar{v}$, then seller t can charge $\underline{v}$ and get a strictly higher profit. ■

A.2 Pure Strategy: Proof of Lemma 1

Lemma 5 $K_1 \cap \left[\bar{k}_2, \bar{k}_1\right] = \left\{\bar{k}_1\right\}$.

Proof: To prove this claim, note that by the definition of $\bar{k}_2$, the buyer type $k \in [\bar{k}_2, \bar{k}_1]$ is guaranteed to trade at or before period 2. Therefore, by choosing a marginal type $k \in [\bar{k}_2, \bar{k}_1]$, seller 1 would sell with probability $1 - F(k)$. The price $p_1(k)$ is such that the marginal type k, who will buy for sure next period, is indifferent between buying now or waiting:

$$k - p_1(k) = \delta\left(k - E\left[p_2\right]\right).$$

Hence, $p_1(k) = (1-\delta)k + \delta E\left[p_2\right]$ and therefore seller 1's problem is

$$\max_k \ (1 - F(k)) \times [(1-\delta)k + \delta E\left[p_2\right]].$$

The derivative of the objective function can be calculated to be

$$-f(k)\left[(1-\delta)\left(k-\frac{1-F(k)}{f(k)}\right)+\delta E[p_2]\right].$$

Since $k-\frac{1-F(k)}{f(k)}$ is strictly increasing by assumption, this derivative is strictly increasing over the interval $\left[\bar{k}_2,\bar{k}_1\right]$. Now since $\bar{k}_1$ maximizes seller 1's profit (or types arbitrarily close to $\bar{k}_1$ if $\bar{k}_1=\sup K_1$ is not achieved by any $k\in K_1$) and $\bar{k}_1$ is interior, it must be that at $k=\bar{k}_1$, this derivative must be 0 and since it is is strictly increasing, it must be negative for any $k\in[\bar{k}_2,\bar{k}_1)$. Hence, no $k\in[\bar{k}_2,\bar{k}_1)$ is optimal. Therefore, $K_1\cap\left[\bar{k}_2,\bar{k}_1\right]=\left\{\bar{k}_1\right\}$. ■

Note that Lemma 5 does not imply that seller 1 must play a pure strategy. It does not rule out the case that $K_1\backslash\left[\bar{k}_2,\bar{k}_1\right]\neq\varnothing$. However, we are able to successively narrow down K_1. This is done in Lemma 1 of the main text.

Lemma 1 *For any* $\tau=1,...,T-1$, $(\cup_{t=1}^{\tau}K_t)\cap[\bar{k}_{\tau+1},\bar{k}_1]=\left\{\bar{k}_1,\bar{k}_2...,\bar{k}_\tau\right\}$.

Proof: The proof is by induction. We proceed in the following steps.

Step 1: First note that $K_1\cap\left[\bar{k}_2,\bar{k}_1\right]=\left\{\bar{k}_1\right\}$. This is what we proved in Lemma 5. This step shows that $\bar{k}_1$ is an isolated point in K_1.

Step 2: Next we argue that for $1\leq\tau+2\leq T$, if

$$(\cup_{t=1}^{\tau}K_t)\cap[\bar{k}_{\tau+1},\bar{k}_1]=\left\{\bar{k}_1,\bar{k}_2...,\bar{k}_\tau\right\}, \tag{16}$$

then

$$\left(\cup_{t=1}^{\tau+1}K_t\right)\cap[\bar{k}_{\tau+2},\bar{k}_1]=\{\bar{k}_1,\bar{k}_2...,\bar{k}_{\tau+1}\}.$$

In words, we want to show inductively that $\bar{k}_t$ is an isolated point in the support of seller t's cutoffs and no seller will ever set a cutoff in the interval $(\bar{k}_t,\bar{k}_{t+1})$. The induction step is illustrated in Figure 4.

From the induction hypothesis, $(\cup_{t=1}^{\tau}K_t)\cap[\bar{k}_{\tau+1},\bar{k}_1]=\left\{\bar{k}_1,\bar{k}_2...,\bar{k}_\tau\right\}$. Recall that $\bar{k}_t'=\sup K_t\backslash\{\bar{k}_t\}$. Take the *smallest* t^* such that $\bar{k}_{t^*}'=\sup\{\bar{k}_t'|t=1,...,\tau+1\}$. That is, $\bar{k}_{t^*}'$ is the highest among the "second highest equilibrium cutoffs" in periods up to $\tau+1$. Note that by the induction hypothesis for any $t\leq\tau$,

$$\bar{k}_{t^*}'\leq\bar{k}_{\tau+1}<\bar{k}_t. \tag{17}$$

If $K_t\backslash\{\bar{k}_t\}=\varnothing$ for all $t=1,...,\tau+1$, then the proof is complete already. Suppose this is not the case. If $\bar{k}_{t^*}'<\bar{k}_{\tau+2}$, then the induction is complete as well. Now suppose $\bar{k}_{t^*}'\geq\bar{k}_{\tau+2}$.

Step 3: We establish the following claims.

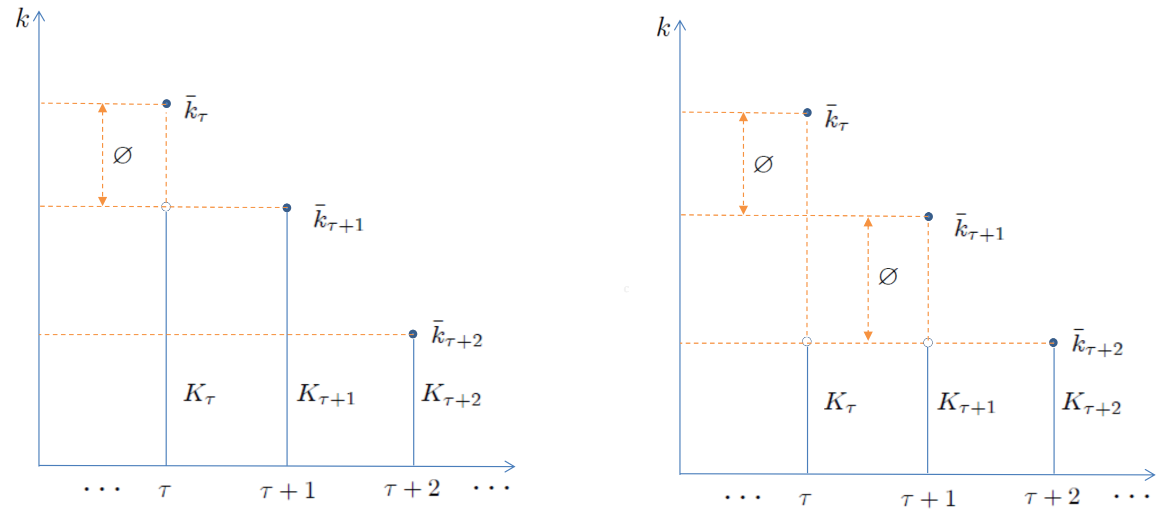

Figure 4: The left panel depicts the induction hypothesis; the right panel depicts the induction step.

Claim 1: There exists $\varepsilon > 0$ such that $(\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon) \bigcap K = \varnothing$. That is, there is no (future or past) cutoff immediately above $\bar{k}'_{t^*}$. Hence, in any period t after any history, buyer types $[\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$ must be either entirely in the support of the posterior or entirely outside of the support of the posterior.

Proof of Claim 1: Note that by (17), we have either $\bar{k}'_{t^*} = \bar{k}_{\tau+1}$ or $\bar{k}'_{t^*} < \bar{k}_{\tau+1}$. By (17), in the former case we have

$$\bar{k}_{\tau+2} < \bar{k}'_{t^*} = \bar{k}_{\tau+1}, \tag{18}$$

and in the latter case, we have

$$\bar{k}_{\tau+2} \leq \bar{k}'_{t^*} < \bar{k}_{\tau+1}. \tag{19}$$

It follows immediately from Lemma 4 that there is no offer (buyer cutoff) within $(\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$ in all periods $t = 1, ..., T$.

Claim 2: There exists $\varepsilon > 0$ such that $(\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon) \bigcap K = \{\bar{k}_{t^*}\}$. That is, there is no (future or past) cutoff in an ε-neighborhood of $\bar{k}_{t^*}$.

Proof of Claim 2: If $t^* \leq \tau$, the claim follows from the induction hypothesis (16). If $t^* = \tau + 1$, the problem arises only when $\bar{k}_{\tau+1} = \bar{k}'_{\tau+1}$ because then $\bar{k}_{\tau+1}$ is not an isolated point. This means that there exists $k^n_{\tau+1} \uparrow \bar{k}_{\tau+1}$. Then it must be that there exists $\bar{t} < \tau + 1$ with equilibrium cutoffs $k^n_{\bar{t}} \in K_{\bar{t}}$ such that $k^n_{\bar{t}} \uparrow \bar{k}_{\tau+1}$; otherwise, seller $\tau + 1$ will not offer both $k^n_{\tau+1}$ and $\bar{k}_{\tau+1}$ following the same line of arguments in Step 1 that establishes Lemma 5. But then we must have $\bar{k}'_{\bar{t}} = \bar{k}'_{t^*}$. Since $\bar{t} < \tau + 1 = t^*$, this contradicts the definition of t^*.

Claim 3: $\bar{k}'_{t^*} \neq \bar{k}_{t^*}$. In addition, $(\bar{k}'_{t^*}, \bar{k}_{t^*}) \bigcap K_t = \varnothing$ for all $t \leq t^*$. That is, $(\bar{k}'_{t^*}, \bar{k}_{t^*})$ includes no past cutoffs.

Proof of Claim 3: First note that $\bar{k}'_{t^*} \neq \bar{k}_{t^*}$ by Claim 2 above. The remaining part of Claim 3 follows from the induction hypothesis.

Claim 4: $(\bar{k}'_{t^*}, \bar{k}_{t^*-1}) \bigcap K_t = \varnothing$ for all $t < t^*$. That is, at the beginning of period t^*, buyer types $(\bar{k}'_{t^*}, \bar{k}_{t^*-1})$ are either entirely in the support of the posterior or entirely outside of the support of the posterior.

Proof of Claim 4: This follows from the induction hypothesis (16) and Claim 3.

From now on, we shall consider seller t^*'s optimization problem. For any buyer type $k \geq \underline{v}$, let $\tau(k)$ be the (random) period at which type k ends up trading if he does not trade at or before time t^*. The distribution of $\tau(k)$ for each k is determined by the equilibrium strategies of sellers $t > t^*$.

Step 4: To target a cutoff type k, seller t^* must choose a price $p(k)$ satisfying the following indifference condition:

$$k - p(k) = \sum_{t=t^*+1}^{\infty} \Pr(\tau(k) = t)\delta^{t-t^*} E\left[k - p_t(k_t) | k_t \leq k, k_t \in K_t\right]. \tag{20}$$

Using the fact that type $k \geq \underline{v}$ must eventually trade in the future, the right-hand side of the above expression can be rewritten as

$$kE[\delta^{\tau(k)-t^*}] - \sum_{t=t^*+1}^{\infty} \Pr(\tau(k) = t)\delta^{t-t^*} E\left[p_t(k_t) | k_t \leq k, k_t \in K_t\right].$$

Denote

$$\mathbf{p}(k) := \sum_{t=t^*+1}^{\infty} \Pr(\tau(k) = t)\delta^{t-t^*} E\left[p_t(k_t) | k_t \leq k, k_t \in K_t\right],$$

and

$$\mathbf{d}(k) := E[\delta^{\tau(k)-t^*}].$$

Hence $\mathbf{p}(k)$ and $\mathbf{d}(k)$ are type k's expected discounted trading price and discounted trading probability from period $t^* + 1$ on, conditional on this type not having traded at or before t^*. Hence, (20) can be rewritten as

$$k - p(k) = k\mathbf{d}(k) - \mathbf{p}(k).$$

Therefore, the cutoff price for type k at period t^* can be written as $p(k) = k(1 - \mathbf{d}(k)) + \mathbf{p}(k)$.

Step 5: We now consider seller t^*'s objective function. If seller t^* targets a cutoff type $k \in [\bar{k}'_{t^*}, \bar{k}_{t^*-1})$, the trading probability can be written in the form of

$$\beta\left(\alpha - F\left(k\right)\right)$$

for some positive number $\alpha \in \left(F\left(\bar{k}_{t^*-1}\right), 1\right]$ and $\beta < 1$. This follows from Claim 4 in Step 3: either all buyer types in the interval $(\bar{k}'_{t^*}, \bar{k}_{t^*-1})$ have traded before t^* or none of them has traded before t^*. Therefore, seller t^*'s payoff by choosing a cutoff $k \in [\bar{k}'_{t^*}, \bar{k}_{t^*-1})$ is

$$R\left(k\right) = \beta\left(\alpha - F\left(k\right)\right)\left[k\left(1 - \mathbf{d}\left(k\right)\right) + \mathbf{p}(k)\right]. \tag{21}$$

Moreover, by Claim 2 in Step 3, if types $\left(\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon\right)$ do not trade at period t^*, they will trade together in the future. Thus we have

$$\mathbf{d}(k) \equiv \mathbf{d}(\bar{k}_{t^*}) \text{ and } \mathbf{p}(k) \equiv \mathbf{p}(\bar{k}_{t^*}) \text{ for all } k \in \left(\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon\right).$$

By Claim 1 in Step 3, we have

$$\mathbf{d}(k) \equiv \mathbf{d}(\bar{k}'_{t^*}) \text{ and } \mathbf{p}(k) \equiv \mathbf{p}(\bar{k}'_{t^*}) \text{ for all } k \in [\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon).$$

In sum, seller t^*'s payoff (21) as a function of k is such that

$$\begin{aligned} R\left(k\right) &= \beta\left(\alpha - F\left(k\right)\right)\left[k\left(1 - \mathbf{d}\left(\bar{k}_{t^*}\right)\right) + \mathbf{p}(\bar{k}_{t^*})\right] \text{ if } k \in (\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon); \\ R\left(k\right) &= \beta\left(\alpha - F\left(k\right)\right)\left[k\left(1 - \mathbf{d}\left(\bar{k}'_{t^*}\right)\right) + \mathbf{p}(\bar{k}'_{t^*})\right] \text{ if } k \in [\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon). \end{aligned}$$

Step 6: Now consider the derivative of seller t^*'s payoff with respect to k. In the case that $k \in (\bar{k}_{t^*} - \varepsilon, \bar{k}_{t^*} + \varepsilon)$,

$$\frac{\partial R(k)}{\partial k} = -f(k)\left[\left(1 - \mathbf{d}\left(\bar{k}_{t^*}\right)\right)\left(k - \frac{\alpha - F\left(k\right)}{f\left(k\right)}\right) + \mathbf{p}(\bar{k}_{t^*})\right].$$

Hence, we have

$$\frac{\partial R(k)}{\partial k} = 0. \tag{22}$$

Note that in the case that $k \in [\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon)$, $R\left(k\right)$ is right-differentiable in k. As a result, for $k \in \left(\bar{k}'_{t^*}, \bar{k}'_{t^*} + \varepsilon\right)$,

$$\frac{\partial R(k)}{\partial k} = -f(k)\left[\left(1 - \mathbf{d}\left(\bar{k}'_{t^*}\right)\right)\left(k - \frac{\alpha - F\left(k\right)}{f\left(k\right)}\right) + \mathbf{p}(\bar{k}'_{t^*})\right]. \tag{23}$$

Let $\frac{\partial R}{\partial k}\left(\bar{k}_{t^*}'\right)$ denote the left-derivative of $R\left(k\right)$ at $k=\bar{k}_{t^*}'$. Then

$$\begin{aligned}
\frac{\partial R}{\partial k}\left(\bar{k}_{t^*}'\right) &= \left(1-\mathbf{d}\left(\bar{k}_{t^*}'\right)\right)\left(\bar{k}_{t^*}'-\frac{\alpha-F\left(\bar{k}_{t^*}'\right)}{f\left(\bar{k}_{t^*}'\right)}\right)+\mathbf{p}(\bar{k}_{t^*}') \\
&= \left(1-\mathbf{d}\left(\bar{k}_{t^*}'\right)\right)\left[\left(\bar{k}_{t^*}'-\frac{\alpha-F\left(\bar{k}_{t^*}'\right)}{f\left(\bar{k}_{t^*}'\right)}\right)+\frac{\mathbf{p}(\bar{k}_{t^*}')}{1-\mathbf{d}\left(\bar{k}_{t^*}'\right)}\right] \\
&< \left(1-\mathbf{d}\left(\bar{k}_{t^*}'\right)\right)\left[\left(\bar{k}_{t^*}-\frac{\alpha-F\left(\bar{k}_{t^*}\right)}{f\left(\bar{k}_{t^*}\right)}\right)+\frac{\mathbf{p}(\bar{k}_{t^*})}{1-\mathbf{d}\left(\bar{k}_{t^*}\right)}\right] \\
&= 0 \text{ (by (22))}.
\end{aligned}$$

Note that the inequality in the previous display follows from (i) the fact that both $\mathbf{p}(k)$ and $\mathbf{d}(k)$ are increasing in k; and (ii) the fact that

$$\bar{k}_{t^*}-\frac{\alpha-F(\bar{k}_{t^*})}{f(\bar{k}_{t^*})}>\bar{k}_{t^*}'-\frac{\alpha-F(\bar{k}_{t^*}')}{f(\bar{k}_{t^*}')}.$$

Fact (i) follows from the definition of $\mathbf{p}(k)$ and $\mathbf{d}(k)$. Fact (ii) follows from Lemma 2.

Step 7: Now we have established $\frac{\partial R}{\partial k}\left(\bar{k}_{t^*}'\right)<0$. Hence $\bar{k}_{t^*}'\notin K_{t^*}$. In particular, there exists $\eta>0$ such that

$$R\left(\bar{k}_{t^*}'+\frac{\varepsilon}{2}\right)>R\left(\bar{k}_{t^*}'\right)+\eta. \tag{24}$$

Now since $\bar{k}_{t^*}'\notin K_{t^*}$, there exists a sequence $k_{t^*}^n\in K_{t^*}$ such that $k_{t^*}^n\uparrow\bar{k}_{t^*}'$. By skimming property, a price acceptable by $k_{t^*}^n$ is also acceptable by $\bar{k}_{t^*}'$. As $k_{t^*}^n$ becomes arbitrarily close to $\bar{k}_{t^*}'$, the probability of sale from targeting $k_{t^*}^n$ becomes arbitrarily close to the probability of sale from targeting $\bar{k}_{t^*}'$, and hence for n large enough,

$$R\left(\bar{k}_{t^*}'\right)>R\left(k_{t^*}^n\right)-\frac{\eta}{2}. \tag{25}$$

It follows from (24) and (25) that for large n,

$$R\left(\bar{k}_{t^*}'+\frac{\varepsilon}{2}\right)>R\left(k_{t^*}^n\right)+\frac{\eta}{2}.$$

Hence, $k_{t^*}^n$ cannot be optimal, a contradiction. This establishes that either $K_t\backslash\{\bar{k}_t\}=\varnothing$ for all $t=1,...,\tau+1$, or otherwise $\bar{k}_{t^*}'<\bar{k}_{\tau+2}$. The induction is therefore complete. ■

A.3 Existence and Uniqueness

Define the following function:

$$k(b,p)=\arg\max_{k\geq p}[F(b)-F(k)][(1-\delta)k+\delta p]. \tag{26}$$

The solution to the maximization problem exists by the continuity of F and is unique by the assumption that F satisfies increasing virtual valuation. Therefore, $k(b,p)$ is a well-defined function. Intuitively, $k(b,p)$ is the current period marginal type that maximizes the seller's revenue given that the next period price is p and the highest type that has not traded so far is b.

Lemma 6 *(1) $k(b,p)$ is continuous. (2) As long as $k(b,p) > p$, it is strictly increasing in b and strictly decreasing in p.*

Proof: (1) follows from the maximum theorem. (2) says that when the next period price is higher, the current seller chooses a lower cutoff (sells to more buyer types) and when the highest remaining type b is higher, he chooses a higher cutoff (sells to fewer types). Note that the first-order condition is given by

$$k' - \frac{F(b) - F(k')}{f(k')} = -\frac{\delta}{1-\delta} p$$

and the claim follows. ∎

Define the sequences $b_0, b_1, ...$ and $p_0, p_1, ...$, inductively, as follows:

$$\begin{aligned} b_0 &= p_0 = \underline{v}; \\ b_s &= \sup \{ b : k(b, p_{s-1}) = b_{s-1} \}; \\ p_s &= (1-\delta) b_s + \delta p_{s-1}. \end{aligned}$$

Intuitively, we have reversed the timeline for the purpose of backward induction. When the game is over, period 0, the largest remaining type is $b_0 = \underline{v}$ and the price that leads to $b_0 = \underline{v}$ (in the previous period) is $p_0 = \underline{v}$. Now b_1 is the largest type such that the game will finish this period if the remaining set of types is $[\underline{v}, b_1]$. Then p_1 is the price that leads to b_1 (in the previous period). Hence, if the remaining set of types is $[\underline{v}, b_s]$, the game will finish in s periods (including the current period), and p_s is the price that leads to the marginal type b_s (in the previous period).

Given (b_{s-1}, p_{s-1}) and $b_{s-1} \geq p_{s-1}$, b_s is (and therefore p_s is) uniquely defined. However, we cannot remove "sup" because if $b_{s-1} = p_{s-1}$, then any $b \in (b_{s-1}, b_s]$ satisfies (26).

Lemma 7 *The set $\{b : k(b, p_{s-1}) = b_{s-1}\}$ is a singleton whenever $b_{s-1} > p_{s-1}$.*

Lemma 8 *(Existence) For each initial belief $b \in (b_s, b_{s+1}]$, there is an equilibrium that ends in exactly $s+1$ periods.*

Proof: By definition of b_1, there is an equilibrium in which the game ends in 1 period if the initial belief is in $(b_0, b_1]$. This establishes the claim for $s = 0$. Now we construct an equilibrium for $s > 1$. For each $b \in (b_0, b_1]$, set $\beta_0(b) = b$ and $\pi_0(b) = \underline{v}$ and for $n = 1, ..., s$, define $\pi_n, \beta_n : (b_0, b_1] \to \mathbb{R}$, inductively, by the following:

$$\pi_n(b) = (1-\delta)\beta_{n-1}(b) + \delta\pi_{n-1}(b) \quad \text{and} \quad k(\beta_n(b), \pi_{n-1}(b)) = \beta_{n-1}(b).$$

Now we claim that β_n, π_n satisfy the following properties:

(1) $\beta_n(b)$ is strictly increasing and continuous,

(2) $\beta_n(b) > \pi_{n-1}(b)$,

(3) $\pi_n(b)$ is continuous and weakly increasing,

(4) $\beta_n(b_0) = b_n$ and $\beta_n(b_1) = b_{n+1}$.

First, we argue that (1)-(4) hold for $n = 1$. Since $b > b_0 = \pi_0(b)$, by Lemma 6, $\beta_1(b)$ is strictly increasing and continuous, establishing property (1). Property (4) follows by definition of the sequence $b_0, b_1, ...$ Property (2) follows because $\beta_1(b)$ is strictly increasing, $b > b_0$ and $\beta_1(b_0) = b_1 \geq b_0 = \pi_0(b)$.

Now assume that (1)-(4) hold for $n = 1, ..., s-1$. We claim that they hold for $n = s$. Note that $\beta_s(b)$ is continuous by induction hypothesis (3) and the continuity of $k(\cdot, \cdot)$. Again, by Lemma 6 and induction hypothesis (3), $\beta_s(b)$ is strictly increasing. So (1) is confirmed. (3) is immediate by definition of $\pi_s(b)$. By the definition of $\beta_s(\cdot)$,

$$\begin{aligned} k(\beta_s(b_0), \pi_{s-1}(b_0)) &= \beta_{s-1}(b_0) = b_{s-1}, \\ k(\beta_s(b_1), \pi_{s-1}(b_0)) &= \beta_{s-1}(b_1) = b_s. \end{aligned}$$

By the definition of b_s and the uniqueness of β_s, we know immediately that

$$\begin{aligned} \beta_s(b_0) &= b_s, \\ \beta_s(b_1) &= b_{s+1}. \end{aligned}$$

Therefore (4) is confirmed. Note that

$$\beta_s(b) > \beta_{s-1}(b) > \pi_s(b).$$

We immediately get property (2) for $n = s$.

So far, we have shown that β_n is a one-to-one and onto map from the interval $(b_0, b_1]$ into $(b_n, b_{n+1}]$. This implies that for any initial belief $b \in (b_s, b_{s+1}]$, there exists a unique $b^* \equiv \beta_s^{-1}(b) \in (b_0, b_1]$. Moreover, it is easy to see that the sequence $\{\beta_{s-1}(b^*), ..., \beta_0(b^*), \underline{v}\}$ form a sequence of equilibrium cutoffs for the game starting with belief b. This establishes the claim. ■

Next, we show that the equilibrium constructed above is the unique equilibrium.

Lemma 9 *For all initial beliefs $b \in (b_s, b_{s+1}]$, in any equilibrium, trade must be completed in exactly $s+1$ periods. Moreover, there is a unique such equilibrium.*

Proof: *Step 1:* We first show that this is true for $s = 0$, i.e., there are no equilibria that last more than 1 period for $b \in (b_0, b_1]$ and there is a unique such equilibrium in which the first seller charges $\underline{v}$.

First, note that there exists b^* very small such that the game ends in one shot. Then consider $b \in (b^*, b^* + \varepsilon]$, where ε is such that for any $v \in [b^*, b_1]$,

$$(F(b) - F(b - \varepsilon))\, b < F(b)\, \underline{v}$$

or

$$\left(1 - \frac{F(b - \varepsilon)}{F(b)}\right) b < \underline{v}.$$

If in equilibrium seller 1 chooses a cutoff in $(b^*, b^* + \varepsilon)$, then it contradicts the choice of ε. If in equilibrium, seller 1 chooses a cutoff in $(\underline{v}, b^*]$, then the game ends in two periods. Then from Equation (26) seller 1's problem is

$$k(b, \underline{v}) = \arg\max_{k \geq \underline{v}} \left[F(b) - F(k)\right]\left[(1 - \delta)\, k + \delta \underline{v}\right].$$

Since the game ends in two periods, we know $k(b, \underline{v}) > \underline{v}$. Therefore, by the monotonicity established in Lemma 6, $k(b, \underline{v})$ is strictly increasing in b. Therefore for $b \leq b_1$, we have $k(b, \underline{v}) \leq k(b_1, \underline{v}) = \underline{v}$. Contradicting the assumption $k(b, \underline{v}) > \underline{v}$. That is, the game must end in one period if $b \in (b^*, b^* + \varepsilon]$.

Now consider $v \in (v^* + \varepsilon, v^* + 2\varepsilon]$. In equilibrium, the period 2 cutoff cannot be in $(\underline{v} + \varepsilon, v^* + 2\varepsilon]$ because of the choice of ε. If the period 2 cutoff is in $(\underline{v}, v^* + \varepsilon]$, then the game ends in two periods. But we can apply the previous argument again to derive a contradiction. The proof for this step is completed by induction.

Suppose it is true for $s = 1, 2, ..., N - 1$ and consider $b \in (b_N, b_{N+1}]$.

Step 2: We first show that the game ends in exactly $N + 1$ periods.

Step 2.1: Suppose there exists an equilibrium that lasts longer than $N + 1$ periods. Then, it must be the case that in this equilibrium, the first seller with initial belief b chooses a cutoff $\tilde{k}_1 > b_N$, because, by the induction hypothesis, for any smaller cutoff the game ends in exactly N additional periods. Similarly, the second seller with initial belief $\tilde{k}_1$ chooses a cutoff level $\tilde{k}_2 > b_{N-1}$ (note that this does not exclude the case where $\tilde{k}_2 > b_N$) and the s^{th} seller with initial belief $\tilde{k}_{s-1}$ chooses $\tilde{k}_s > b_{N-s+1}$ so that the game lasts more than $N+1$ periods. Therefore, the price that the second seller with initial belief $\tilde{k}_1$ charges is strictly greater than

$$(1 - \delta)(b_{N-1} + \delta b_{N-2} + ... + \delta^{N-2} b_1) + \delta^{N-1} \underline{v}.$$

On the other hand, we also know, by Lemma 8, that there is an equilibrium that lasts exactly $N + 1$ periods. Let k_s^* be the cutoff sequence of that equilibrium. Notice that $k_s^* \leq b_{N+1-s}$.

Therefore, the price charged by the second period seller with initial belief k_1^* is at most

$$(1-\delta)(b_{N-1}+\delta b_{N-2}+...+\delta^{N-2}b_1)+\delta^{N-1}\underline{v}.$$

But then, the first seller in the candidate equilibrium chooses a higher cutoff than the first seller in the equilibrium of Lemma 8, even though both of these sellers have the same initial belief and the second period price is less in the latter equilibrium. Since the optimal cutoff is decreasing in the continuation price, this is a contradiction by Lemma 6.

Step 2.2: Now, suppose there is an equilibrium that lasts N periods or less.

Suppose, first, that the cutoff $\hat{k}_1$ that the first seller with belief b chooses in equilibrium is less than b_{N-1}. Thereafter, there is a unique continuation equilibrium in which for all s the cutoff chosen by the s^{th} seller with belief $\hat{k}_{s-1}$ is at most b_{N-s}. Therefore, the price charged by the second seller is at most

$$(1-\delta)(b_{N-2}+\delta b_{N-3}+...+\delta^{N-2}b_1)+\delta^{N-1}\underline{v}.$$

On the other hand, the cutoff k_s^* chosen by seller s in the equilibrium of Lemma 8 is strictly greater than b_{N-s}. And, therefore, the price is strictly above

$$(1-\delta)(b_{N-2}+\delta b_{N-3}+...+\delta^{N-2}b_1)+\delta^{N-1}\underline{v}.$$

But this is a contradiction since $k(b,p)$ is decreasing in p.

Now suppose that $\hat{k}_1 > b_N$. Let s be the first period when the cutoff $\hat{k}_s \leq b_N$. Then it must be that $\hat{k}_s \leq b_{N-s}$, because for any $k \in (b_{N-s}, b_N]$, there is a unique continuation equilibrium that lasts at least $N-s+1$ periods. Now, consider the equilibrium constructed in Lemma 8, starting from initial belief $\beta_{N-s}(\hat{k}_s)$. Then, by construction, the seller with this belief chooses $\hat{k}_s$. Moreover, by the induction hypothesis, the continuation of this equilibrium coincides with the continuation of the other equilibrium where $\hat{k}_{s-1}$ chooses $\hat{k}_s$. This implies that the next period price is the same in both equilibria. Call this price p. Note that $\hat{k}_{s-1} > b_N > \beta_{N-s}(\hat{k}_s)$. But this is a contradiction since $k(\hat{k}_{s-1},p) > k(\beta_{N-s}(\hat{k}_s),p)$.

Step 3: Step 2 establishes that for $b \in (b_N, b_{N+1}]$, all equilibria last exactly $N+1$ periods. We show that this equilibrium is unique – the one we constructed in Lemma 8.

Suppose by contradiction there is another equilibrium (in addition to the one constructed in the proof of Lemma 8) that lasts exactly $N+1$ periods. Let k^* be the first period cutoff of the equilibrium constructed in the proof of Lemma 8. Then, $k^* \leq b_N$. Let k' be the first cutoff of the other equilibrium. Then it must be that $k^* \neq k'$ because there is a unique N period equilibrium following cutoff k^* by the induction hypothesis.

Now, if $k' \leq b_N$, it must be that $k' > b_{N-1}$, since otherwise the equilibrium lasts at most $N-1$ periods. Suppose, w.l.o.g., that $k^* > k'$. Then it must be the case that the second period price in equilibrium of Lemma 8 is higher than the second period price following k'. This is because, in the unique continuation equilibrium, all cutoffs are increasing in the initial belief, since the functions $\beta_s(\cdot)$ are increasing; and because, after each of these cutoffs, the equilibrium lasts exactly N additional periods. But this leads to a contradiction since $k(b, p)$ is decreasing in p.

Now, suppose $k^* > b_N$. Now, we use an argument similar to the one used to establish that all equilibria last at least $N + 1$ periods. Let s be the first period when the cutoff $\hat{k}_s \leq b_N$. Then it must be that $b_{N-s} \leq \hat{k}_s \leq b_{N-s+1}$, because that is the only way that the equilibrium will have $N - s + 1$ additional periods. Now, consider the equilibrium constructed in Claim 1, starting from initial belief $\beta_{N-s}(\hat{k}_s)$. Then, by construction, the seller with this belief chooses $\hat{k}_s$. Moreover, by the induction hypothesis, the continuation of this equilibrium coincides with the continuation of the other equilibrium where $\hat{k}_{s-1}$ chooses $\hat{k}_s$. This implies that the next period price is the same in both equilibria. Call this price p. Note that $\hat{k}_{s-1} > b_N > \beta_{N-s}(\hat{k}_s)$. But this is a contradiction since $k(\hat{k}_{s-1}, p) > k(\beta_{N-s}(\hat{k}_s), p)$. ■

B Proof of Theorem 2

B.1 Preliminary Results

Notice that, under the transparent regime, the second period price (on or off the equilibrium path) can depend only on the first period price, as this is the only observable history. Also, following an off-equilibrium first period price, seller 2 may play a mixed strategy. Let $\hat{p}_2^{TR}(p)$ be the expected second period price if the first period seller in the transparent regime chooses p.

We first establish some preliminary results. The next lemma establishes that the profit maximization prices of any seller is non-decreasing in the highest type $\bar{k}$ that he believes to be remaining, regardless of the continuation play. Note that the solution to this maximization problem may not be unique. Therefore, the monotonicity claim requires the definition of an ordering of sets. The appropriate definition in this context is as follows:

Definition 1 *Let $X, Y \subset \mathbb{R}$. We say that X is greater than Y if and only if for all $x \in X$ and $y \in Y$, $x \geq y$.*

Lemma 10 *The solution to the problem of choosing p to maximize $[F(\bar{k}) - F(k(p))]p$ is non-decreasing in $\bar{k}$.*

Proof: This follows because the objective function has increasing differences in $\bar{k}$ and p. ■

Clearly, for any p, there exists at most a unique k that satisfies

$$p = (1-\delta)k + \delta \hat{p}_2^{TR}(p),$$

as long as $\delta < 1$. Let $k_1^{TR}(p)$ be defined by this indifference condition. Also let $k_1^{NTR}(p)$ be defined by

$$p = (1-\delta)k_1^{NTR}(p) + \delta p_2^{NTR}.$$

Lemma 11 *$k_1^{TR}(p)$ is non-decreasing in p.*

Proof: Take $p > p'$ and suppose that $k_1^{TR}(p) < k_1^{TR}(p')$. Then, by Lemma 10

$$p = (1-\delta)k^{TR}(p) + \delta \hat{p}_2(p) \leq (1-\delta)k^{TR}(p') + \delta \hat{p}_2(p') = p',$$

a contradiction. ■

The next lemma shows that in the transparent regime, a deviation by the first seller to a higher price weakly increases the expected price in the second period.

Lemma 12 *If $p_1^{TR} < p$, then $\hat{p}_2^{TR}(p_1^{TR}) \leq \hat{p}_2^{TR}(p)$.*

Proof: Take $p > p_1^{TR}$. Then Lemma 11 implies that $k_1^{TR}(p_1^{TR}) \leq k_1^{TR}(p)$. Moreover, $k_1^{TR}(p_1^{TR}) = k_1^{TR}(p)$ contradicts the optimality of p_1^{TR}. Therefore, $k_1^{TR}(p_1^{TR}) < k_1^{TR}(p)$. Then the claim follows from Lemma 10. ■

B.2 Proof of Theorem 2

The proof is by induction on the maximum number of periods that it takes for trade to be completed. Let $T^i, i = TR, NTR$ be the last period during which trade takes place with positive probability in regime i on the given equilibrium path. First, if $\max\{T^{TR}, T^{NTR}\} = 1$, then observability does not play a role and therefore the prices in the two regimes are the same, and equal to $\underline{v}$. Hence, the claim is vacuously satisfied.

Assume that the claim is true when $\max\{T^{TR}, T^{NTR}\} = 1, 2, ..., \tau$. Consider the case where $\max\{T^{TR}, T^{NTR}\} = \tau + 1$.

Lemma 13 *Let p_1^{TR} be any price in the support of seller 1's strategy in the transparent regime. Then, $\hat{p}_2^{TR}(p_1^{TR}) \geq p_2^{NTR}$. That is, the expected second period price in the transparent regime following any equilibrium path history is no less than the second period price in the non-transparent regime.*

Proof: Suppose $\hat{p}_2^{TR}(p_1^{TR}) < p_2^{NTR}$. We first claim that $k_1^{TR}(p_1^{TR}) < k_1^{NTR}(p_1^{NTR})$. To see this, note that from the second period onward, all trade takes place in at most τ periods in either regime, and hence the induction hypothesis applies. Now, by the induction hypothesis and Lemma 10, if $k_1^{TR}(p_1^{TR}) \geq k_1^{NTR}(p_1^{NTR})$, then $\hat{p}_2^{TR}(p_1^{TR}) \geq p_2^{NTR}$ which establishes the claim. For the rest of the proof, we let $k_1^i \equiv k_1^i(p_1^i), i = NTR, TR$. That is, k_1^i is the *equilibrium* marginal type that purchases in period 1 in regime i.

Let $p_1^i(k)$ be the supremum of prices in the inverse of $k_1^i(p)$; that is for each k, $p_1^i(k)$ is the supremum of prices at which seller 1 in regime i can choose to sell to the marginal type k in period 1. Clearly, seller 1's payoff in regime i cannot be strictly less than $(1-F(k))p_1^i(k)$ for any k.

Now, since p_1^{NTR} is seller 1's unique optimal price in the non-transparent regime, we have that

$$(1-F(k_1^{TR}))p_1^{NTR}(k_1^{TR}) < (1-F(k_1^{NTR}))p_1^{NTR}(k_1^{NTR})$$

which can be re-arranged as

$$\frac{1-F(k_1^{TR})}{1-F(k_1^{NTR})} < \frac{p_1^{NTR}(k_1^{NTR})}{p_1^{NTR}(k_1^{TR})} \Leftrightarrow \frac{F(k_1^{NTR})-F(k_1^{TR})}{1-F(k_1^{NTR})} < \frac{p_1^{NTR}(k_1^{NTR})-p_1^{NTR}(k_1^{TR})}{p_1^{NTR}(k_1^{TR})}.$$

In words, for seller 1 in the non-transparent regime, an increase from k_1^{TR} to k_1^{NTR} of the cutoff type that he trades with decreases the probability of trade by a factor of $\frac{F(k_1^{NTR})-F(k_1^{TR})}{1-F(k_1^{NTR})}$, but is accompanied by a percentage increase in price of $\frac{p_1^{NTR}(k_1^{NTR})-p_1^{NTR}(k_1^{TR})}{p_1^{NTR}(k_1^{TR})}$, which is larger than this factor. Therefore, such a switch is desirable. The rest of the proof argues that the same switch to the cutoff k_1^{NTR} from cutoff k_1^{TR} strictly increases the payoff of seller 1 by showing that in the transparent regime, this switch is accompanied by an even larger percentage increase in price than in the non-transparent regime. This contradicts the optimality of p_1^{TR}.

Formally, we compare

$$\frac{p_1^{NTR}(k_1^{NTR})-p_1^{NTR}(k_1^{TR})}{p_1^{NTR}(k_1^{TR})}$$

to

$$\frac{p_1^{TR}(k_1^{NTR})-p_1^{TR}(k_1^{TR})}{p_1^{TR}(k_1^{TR})}.$$

Note that

$$p_1^{TR}(k_1^{TR}) = p_1^{TR} = (1-\delta)k_1^{TR} + \delta\hat{p}_2(p_1^{TR}) < (1-\delta)k_1^{TR} + \delta p_2^{NTR} = p_1^{NTR}(k_1^{TR}).$$

Here, the inequality follows from the supposition that $\hat{p}_2(p_1^{TR}) < p_2^{NTR}$. Moreover,

$$\begin{aligned} p_1^{NTR}(k_1^{NTR}) - p_1^{NTR}(k_1^{TR}) &= (1-\delta)(k_1^{NTR} - k_1^{TR}) \\ &\leq (1-\delta)(k_1^{NTR} - k_1^{TR}) + \delta(\hat{p}_2^{TR}(p_1^{TR}(k_1^{NTR})) - \hat{p}_2^{TR}(p_1^{TR})) \\ &= p_1^{TR}(k_1^{NTR}) - p_1^{TR}(k_1^{TR}). \end{aligned}$$

Here the inequality follows by Lemma 12, because $k_1^{TR}(p_1^{TR}) < k_1^{NTR}(p_1^{NTR})$ and therefore, $p_1^{TR}(k_1^{TR}) \leq p_1^{TR}(k_1^{NTR})$. But then we have

$$\frac{F(k_1^{NTR}) - F(k_1^{TR})}{1 - F(k_1^{NTR})} < \frac{p_1^{TR}(k_1^{NTR}) - p_1^{TR}(k_1^{TR})}{p_1^{TR}(k_1^{TR})}$$

which contradicts the optimality of k_1^{TR} for seller 1 in the transparent regime. ■

Next we argue that the first period price of the transparent regime is larger than the first period price in the non-transparent regime:

Lemma 14 *Let p_1^{TR} be any realized first period price in the transparent regime. Let p_1^{NTR} be the first period equilibrium price of the non-transparent regime. Then, $p_1^{TR} \geq p_1^{NTR}$.*

Proof: For a contradiction suppose that there exists p_1^{TR} in the support of seller 1 in the transparent regime such that $p_1^{TR} < p_1^{NTR}$.

Since seller 1 in the non-transparent regime has a unique optimal strategy, we have

$$\frac{1 - F(k_1^{NTR}(p_1^{NTR}))}{1 - F(k_1^{NTR}(p_1^{TR}))} > \frac{p_1^{TR}}{p_1^{NTR}} \Leftrightarrow \frac{F(k_1^{NTR}(p_1^{NTR})) - F(k_1^{NTR}(p_1^{TR}))}{1 - F(k_1^{NTR}(p_1^{TR}))} < \frac{p_1^{NTR} - p_1^{TR}}{p_1^{NTR}}. \tag{27}$$

Now we compare

$$\frac{F(k_1^{NTR}(p_1^{NTR})) - F(k_1^{NTR}(p_1^{TR}))}{1 - F(k_1^{NTR}(p_1^{TR}))} = \frac{F(k_1^{NTR}(p_1^{TR}) + \Delta^{NTR}) - F(k_1^{NTR}(p_1^{TR}))}{1 - F(k_1^{NTR}(p_1^{TR}))}$$

to

$$\frac{F(k_1^{TR}(p_1^{NTR})) - F(k_1^{TR}(p_1^{TR}))}{1 - F(k_1^{TR}(p_1^{TR}))} = \frac{F(k_1^{TR}(p_1^{TR}) + \Delta^{TR}) - F(k_1^{TR}(p_1^{TR}))}{1 - F(k_1^{TR}(p_1^{TR}))},$$

where $\Delta^i \equiv k_1^i(p_1^{NTR}) - k_1^i(p_1^{TR})$, $i = TR, NTR$.

Now we observe that:

1. $k_1^{NTR}(p_1^{TR}) \geq k_1^{TR}(p_1^{TR})$. This is because

$$p_1^{TR} = (1-\delta)k_1^{NTR}(p_1^{TR}) + \delta p_2^{NTR} = (1-\delta)k_1^{TR}(p_1^{TR}) + \delta \hat{p}_2^{TR}(p_1^{TR})$$

   and by Lemma 13, $p_2^{NTR} \leq \hat{p}_2^{TR}(p_1^{TR})$.

2. $\Delta^{NTR} \geq \Delta^{TR}$. This is because

$$\begin{aligned} k_1^{NTR}(p_1^{NTR}) - k_1^{NTR}(p_1^{TR}) &= \frac{p_1^{NTR} - p_1^{TR}}{1-\delta} \\ &\geq \frac{p_1^{NTR} - p_1^{TR} - (\hat{p}_2^{TR}(p_1^{NTR}) - \hat{p}_2^{TR}(p_1^{TR}))}{1-\delta} \\ &= k_1^{TR}(p_1^{NTR}) - k_1^{TR}(p_1^{TR}). \end{aligned}$$

   Here the inequality follows from Lemma 12.

Then, we have

$$\begin{aligned}\frac{F(k_1^{NTR}(p_1^{TR})+\Delta^{NTR})-F(k_1^{NTR}(p_1^{TR}))}{1-F(k_1^{NTR}(p_1^{TR}))} &\geq \frac{F(k_1^{NTR}(p_1^{TR})+\Delta^{TR})-F(k_1^{NTR}(p_1^{TR}))}{1-F(k_1^{NTR}(p_1^{TR}))} \\ &\geq \frac{F(k_1^{TR}(p_1^{TR})+\Delta^{TR})-F(k_1^{TR}(p_1^{TR}))}{1-F(k_1^{TR}(p_1^{TR}))},\end{aligned}$$

where the first inequality follows from the second observation and the second inequality follows from the first observation together with the assumption of increasing hazard rate. Combining this with (27), we get

$$\frac{F(k_1^{TR}(p_1^{NTR}))-F(k_1^{TR}(p_1^{TR}))}{1-F(k_1^{TR}(p_1^{TR}))} < \frac{p_1^{NTR}-p_1^{TR}}{p_1^{NTR}} \Leftrightarrow \frac{1-F(k_1^{TR}(p_1^{NTR}))}{1-F(k_1^{TR}(p_1^{TR}))} > \frac{p_1^{TR}}{p_1^{NTR}},$$

which contradicts the optimality of p_1^{TR} for seller 1 in the transparent regime. ■

We complete the proof of Theorem 2 with the following lemma:

Lemma 15 *Assume that* $\max\{T^{TR}, T^{NTR}\} = \tau + 1$. *Then for any* t, $p_t^{TR} \geq p_t^{NTR}$.

Proof: The proof is by induction. We have already shown that $p_1^{TR} \geq p_1^{NTR}$. Take $t' \leq \tau + 1$ and assume that for all $t < t'$, the claim is true. Suppose $p_{t'}^{TR} < p_{t'}^{NTR}$. This is only possible if $k_{t'-1}^{TR}(p_{t'-1}^{TR}) < k_{t'-1}^{NTR}(p_{t'-1}^{NTR})$, because otherwise, by the induction hypothesis and by Lemma 10, we would have $p_{t'}^{TR} \geq p_{t'}^{NTR}$. But then,

$$p_{t'-1}^{TR} = (1-\delta)k_{t'-1}^{TR}(p_{t'-1}^{TR}) + \delta p_{t'}^{TR} < (1-\delta)k_{t'-1}^{NTR}(p_{t'-1}^{NTR}) + \delta p_{t'}^{NTR},$$

a contradiction. This completes the proof of the theorem. ■

C Proof of Theorem 3

Let $\{k_t^{TR}\}$ be a realization of equilibrium cutoffs in the transparent regime and $\{k_t^{NTR}\}$ be the equilibrium sequence of cutoffs in the non-transparent regime with the convention that $k_t^i = \underline{v}$ for $t > T^i$ where T^i is the latest period such that trade takes place with positive probability (along the realized path). Define two random variables, X and Y, with the following cumulative distributions:

$$Prob(x \leq k) = \delta^{\tau^{TR}(k)},$$

where $\tau(k)$ is the unique number that satisfies $k \in [k_{\tau^{TR}(k)}, k_{\tau^{TR}(k)-1})$ and

$$Prob(y \leq k) = \delta^{\tau^{NTR}(k)},$$

where $\tau(k)$ is the unique number that satisfies $k \in [k_{\tau^{NTR}(k)}, k_{\tau^{NTR}(k)-1})$.

In words, the support of X is the equilibrium cutoffs in the transparent regime, whereas the support of Y is the equilibrium cutoffs in the non-transparent regime. The marginal types trading at time t or earlier in each regime have a total probability of δ^{t-1} under the relevant random variable.

Lemma 16 *X second order stochastically dominates Y. That is,*

$$\forall k : \int_{\underline{v}}^{k} Prob(x \leq \tilde{k})d\tilde{k} \leq \int_{\underline{v}}^{k} Prob(y \leq \tilde{k})d\tilde{k}.$$

Proof:

$$\int_{\underline{v}}^{k} Prob(x \leq \tilde{k})d\tilde{k} = \sum_{t=\tau^{TR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{TR})$$

and

$$\int_{\underline{v}}^{k} Prob(y \leq \tilde{k})d\tilde{k} = \sum_{t=\tau^{NTR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{NTR})$$

First assume that $\tau^{TR}(k) \geq \tau^{NTR}(k)$. Then,

$$\begin{aligned}
& \sum_{t=\tau^{NTR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{NTR}) - \sum_{t=\tau^{TR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{TR}) \\
= & \sum_{t=\tau^{TR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{TR} - k_t^{NTR}) + \sum_{t=\tau^{NTR}(k)}^{\tau^{TR}(k)-1} (1-\delta)\delta^{t-1}(k - k_t^{NTR}) \\
\geq & \sum_{t=\tau^{TR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{TR} - k_t^{NTR}) \\
\geq & 0
\end{aligned}$$

where the first inequality is by the fact that $k \geq k_t^{NTR}$ for all $t \geq \tau^{NTR}(k)$ by the definition of the latter, and the last inequality is due to the price ranking. Now assume that $\tau^{TR}(k) < \tau^{NTR}(k)$. Then,

$$\begin{aligned}
& \sum_{t=\tau^{NTR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{NTR}) - \sum_{t=\tau^{TR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k - k_t^{TR}) \\
= & \sum_{t=\tau^{NTR}(k)}^{\infty} (1-\delta)\delta^{t-1}(k_t^{TR} - k_t^{NTR}) - \sum_{t=\tau^{TR}(k)}^{\tau^{NTR}(k)-1} (1-\delta)\delta^{t-1}(k - k_t^{TR})
\end{aligned}$$

$$
\begin{aligned}
&\geq \sum_{t=\tau^{NTR}(k)}^{\infty}(1-\delta)\delta^{t-1}(k_t^{TR}-k_t^{NTR}) - \sum_{t=\tau^{TR}(k)}^{\tau^{NTR}(k)-1}(1-\delta)\delta^{t-1}(k_t^{NTR}-k_t^{TR}) \\
&= \sum_{t=\tau^{TR}(k)}^{\infty}(1-\delta)\delta^{t-1}(k_t^{TR}-k_t^{NTR}) \\
&\geq 0
\end{aligned}
$$

where the first inequality follows because by definition of $\tau^{NTR}(k)$ for $t < \tau^{NTR}(k)$ we have $k \leq k_t^{NTR}$ and the last inequality follows from the price ranking. ■

The proof of Theorem 3 immediately follows from the following lemma:

Lemma 17 *If F is concave then,*

$$\sum_{t=1}^{\infty}\delta^{t-1}F(k_t^{TR}) \geq \sum_{t=1}^{\infty}\delta^{t-1}F(k_t^{NTR}).$$

Proof: Notice that for the random variables defined above

$$Prob(x = k_t^{TR}) = Prob(y = k_t^{NTR}) = \delta^{t-1} - \delta^t = (1-\delta)\delta^{t-1}.$$

Then the left- and right-hand sides of the above inequality are the expectation of $F(x)/(1-\delta)$ and the expectation of $F(y)/(1-\delta)$, respectively. Then the claim follows by second-order stochastic dominance. ■

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